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New generalised height-diameter models for the birch stands in European Russia

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Abstract

When measuring in a forest inventory, height and diameter at breast height are basic variables. Generalised models do not require measuring tree heights, and the number of measurements is minimal. However, the opinions of researchers differ in both the number of variables included in the model and in the number of parameters. The purpose of this study was to obtain 24 new generalised height-diameter models based on simple ones, compare them with 9 generalised models selected from other studies, and develop an appropriate height-diameter model for birch in the European part of Russia. The article shows that even in simple cases, there is a wide variety of options for generalised models. Moreover, models with three independent variables may be necessary and sufficient. These are the diameter at breast height, quadratic diameter at breast height, and the mean height. The performance statistics showed that modified power function is the most suitable and, therefore, it is recommended for predicting the height-diameter relationships for birch trees in this study area. The predicting variables for applying developed generalised models to estimate total tree height require less sampling effort. They derive from conventional forest inventory data which cuts costs and saves time during fieldwork.

Keywords: generalised model, height-diameter relationship, total tree height, diameter at breast height, birch stand, European Russia

Introduction

Height and diameter at breast height are rudimentary/primary measurement variables that are measured in a forest inventory. For example, they are used to estimation the volume and biomass of trunks and estimating tree growth (Adame et al. 2008, Picard et al. 2012, Gomez-Garcia et al. 2014, Goussanou et al. 2016). Measuring the diameter at breast height of a tree is accurate and straightforward (Ferraz-Filho ae al. 2018) whereas measuring the height of a tree is an expensive and time-consuming process (Adame et al. 2008, Mehtätalo et al. 2015). Consequently, only heights of subsamples of trees are measured. Height-diameter models are often used to estimate heights of trees with diameters measured (Sánchez-González et al. 2007, Lei at al. 2009, Ogana et al. 2020).

The relationship between height and diameter is complex nonlinear one. Therefore, so it is challenging to describe it with linear models (Adamec and Drápela 2015, Chai et al. 2018). Many models have been developed (Lei et al. 2009, Ahmadi and Alavi 2016, Liu et al. 2017). Simple models describe the relationship between height and diameter at the local level. Usually, two-parameter and three-parameter models stand out amongst the simplest models (Mehtätalo et al. 2015, Sharma et al. 2016, Lebedev and Kuzmichev 2020). The two-parameter models are referable (Mehtätalo et al. 2015, Sharma et al. 2016). However, from a biological viewpoint, three-parameter S-shaped curves are superior because they can convey more accurately the relationship between height and diameter for fine trees (Yuancai and Parresol 2001).

In practice, the generalised models are an alternative to the simple models (Adamec 2015). They do not require measuring tree heights, and they require minimal measurements. Additionally, to the diameter at breast height, generalised models may include quadratic mean diameter, dominant diameter, average height, dominant height, stand basal area, tree number and age (Sonmez 2009, Haruni et al. 2010, Ahmadi and Alavi 2016, Santiago-García et al. 2020). Many generalised models include the dominant diameter and dominant height as predictors. These indicators are not common in forest inventory in Russia and the newly independent countries of the fSU. Despite the importance of height-diameter models in forest growth and yield prediction systems and the long time over which these models have existed for different regions Europe, relatively not that many works on height-diameter models for birch stands in Russian regions has been published. Therefore, the development of generalised models including the quadratic mean diameter and the average height is relevant here. The purpose of this study was to obtain 24 new generalised height-diameter models based on simple models, compare them with 9 generalised models selected from other studies, and develop an appropriate height-diameter model for birch stands in European Russia.

Materials and methods

Data used in this study were collected from 23 sample plots (from 0.2 to 0.5 ha in area) established in the Forest Experimental District, Russian State Agrarian University – Moscow Timiryazev Agricultural Academy. The age of the stands, in which the model trees were measured, was from 10 to 85 years. The average diameter was from 3 to 30 cm, and the average height was from 6 to 27 m. In the experimental plots, 35 to 153 trees were measured. The study area mainly consists of mixed and even-aged forests dominated by pine, larch, birch, oak and linden. The climate is moderately continental. The predominant soils are sod-podzolic (Dubenok et al. 2020). In the herbaceous layer *Galeobdolon luteum* Huds., *Aegopodium podagraria* L., *Geum urbanum* L., *Stellaria media* (L.) Vill., *S. holostea* L., *Luzula pilosa* (L.) Willd., *Dryopteris carthusiana* (Vill.) H.P. Fuchs, *Calamagrostis arundinacea* (L.) Roth, *Lamium album* L., *Milium effusum* L. and others prevail.

For each sample plot diameters and heights of all trees were measured. A total of 2,201 individual tree height-diameter measurements were available for this study. For analysis, the data was divided into fitting and validation samples in a 7:3 ratio. Table 1 shows the mean, minimum and maximum values, and standard deviations of the stand variables. The fitting data was obtained from 1540 individual trees and covers a wide range of tree sizes with diameters ranging from 0.5 to 42.8 cm and tree heights from 2.0 to 28.7 m. The validation data was obtained from 661 individual trees with diameters ranging from 0.7 to 42.1 cm

Table 1. Descriptive statistics for 2201 sample trees

Variable	*Mean*	*Min*	*Max*	*SD*
Fitting data (No. of trees = 1,540)				
DBH (cm)	12.2	0.5	42.8	6.6
h (m)	14.2	2.0	28.7	4.9
D_q (cm)	12.4	2.9	29.3	5.2
H (m)	14.2	5.2	26.1	4.2
Validation data (No. of trees = 661)				
DBH (cm)	12.3	0.7	42.1	6.7
h (m)	14.3	2.5	28.4	4.8
D_q (cm)	12.4	2.9	29.3	5.2
H (m)	14.2	5.2	26.1	4.2

* Note: DBH is the diameter at breast height, h is the tree height, D_q is the quadratic diameter at breast height in each plot, and H is the mean height in each plot

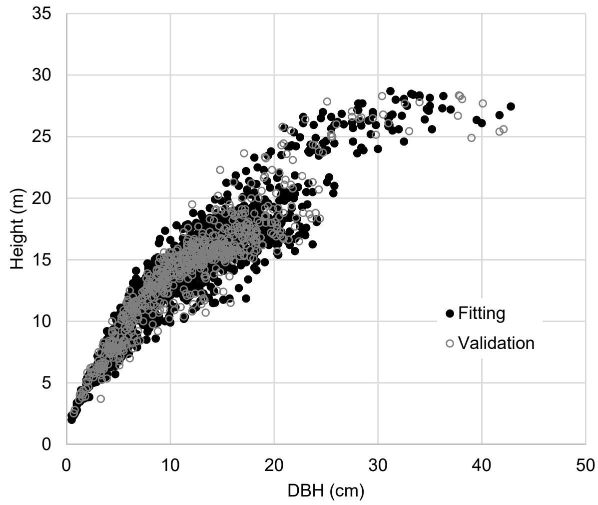

Figure 1. Scatter plots of tree height against diameter at breast height (DBH) of trees for the fitting and the validation data sets

and tree heights from 2.5 to 28.4 m. Scatter plots of tree diameter and height data for the datasets are also illustrated (Figure 1).

In developing the generalised height-diameter models, the simple models were selected from other studies (El Mamoun et al. 2013, Mehtätalo et al. 2015, Hassanzad Navroodi et al. 2016, Liu et al. 2017, Lebedev and Kuzmichev 2020, Ogana et al. 2020). For this study, 12 two-parameter models and 12 three-parameter models were chosen. Four-parameter models were not included in this study since they are more likely to be over-parameterised thereby resulting in instability of the estimates (Fang and Bailey 1998).

When developing generalised models based on simple models, the predictors were diameter at breast height, quadratic diameter at breast height and average height. A generalised model for diameter at breast height equal to a quadratic diameter at breast height should return a height value equal to the average height. As a result, 24 generalised models (M1-M24) which satisfy this condition were obtained. New generalised models contain either 2 or 4 parameters. Simple models and generalised ones are given in Table 2.

Generalised models with predictors of diameter at breast height, quadratic diameter at breast height and mean height were selected from other studies (Table 3) to compare with 24 new models. Selected models (L1-L9) contain from 2 to 10 parameters. Models L5 and L6 are linear and they were designed for young black spruce (*Picea mariana* (Mill.) Britt., E.E. Sterns et Poggenb.) and jack pine (*Pinus banksiana* Lamb.) plantations. The other models are suitable for stands of different ages.

The nonlinear least-squares method was used to fit functions. The trust region reflective algorithm and the dogleg algorithm with rectangular trust regions were used to optimize the objective function. To select models that better describe the relationship between heights and diam-

Table 2. Simple and generalised height-diameter models

ID	Simple model	Generalised model
M1	$h = 1.3 + b_1 DBH^{b_2}$	$h = 1.3 + (H-1.3)\left({DBH}/{D_q}\right)^{a_1+a_2D_q}$
M2	$h = 1.3 + \left(\frac{DBH}{b_1 + b_2 DBH}\right)^2$	$h = 1.3 + (H-1.3)\left(\frac{{DBH}/{D_q}}{1-(a_1+a_2D_q)+(a_1+a_2D_q)\,{DBH}/{D_q}}\right)^2$
M3	$h = 1.3 + \frac{b_1 DBH}{b_2 + DBH}$	$h = 1.3 + (H-1.3)\left(\frac{(1+a_1+a_2D_q)\,{DBH}/{D_q}}{a_1+a_2 * D_q + {DBH}/{D_q}}\right)$
M4	$h = 1.3 + b_1\left(\frac{DBH}{1+DBH}\right)^{b_2}$	$h = 1.3 + \left(\frac{H-1.3}{0.5^{a_1+a_2D_q}}\right)\left(\frac{{DBH}/{D_q}}{\left(1+{DBH}/{D_q}\right)}\right)^{a_1+a_2D_q}$
M5	$h = 1.3 + \frac{b_1 DBH}{(1+DBH)^{b_2}}$	$h = 1.3 + (H-1.3)\left(\frac{2^{a_1+a_2D_q}\,{DBH}/{D_q}}{\left(1+{DBH}/{D_q}\right)^{a_1+a_2D_q}}\right)$
M6	$h = 1.3 + b_1(1-\exp(-b_2 DBH))$	$h = 1.3 + \left(\frac{H-1.3}{1-\exp(-(a_1+a_2D_q))}\right)\left(1-\exp\left(-(a_1+a_2D_q)\,{DBH}/{D_q}\right)\right)$
M7	$h = 1.3 + \exp\left(b_1 + \frac{b_2}{DBH+1}\right)$	$h = 1.3 + (H-1.3)\exp\left(-\frac{a_1+a_2D_q}{2} + \frac{a_1+a_2D_q}{{DBH}/{D_q}+1}\right)$
M8	$h = 1.3 + \frac{b_1 DBH}{(DBH+1)+b_2 DBH}$	$h = 1.3 + \frac{(H-1.3)(2+a_1+a_2D_q)\,{DBH}/{D_q}}{{DBH}/{D_q}+1+(a_1+a_2D_q)\,{DBH}/{D_q}}$
M9	$h = 1.3 + b_1 DBH \exp(-b_2 DBH)$	$h = 1.3 + \left(\frac{H-1.3}{\exp(-(a_1+a_2D_q))}\right)\exp\left(-(a_1+a_2D_q)\,{DBH}/{D_q}\right)$
M10	$h = 1.3 + b_1 \exp\left(\frac{b_2}{DBH}\right)$	$h = 1.3 + \left(\frac{H-1.3}{\exp(a_1+a_2D_q)}\right)\exp\left(\frac{a_1+a_2D_q}{{DBH}/{D_q}}\right)$
M11	$h = 1.3 + b_1(\ln(1+DBH))^{b_2}$	$h = 1.3 + \left(\frac{H-1.3}{(\ln 2)^{a_1+a_2D_q}}\right)\left(\ln\left(1+{DBH}/{D_q}\right)\right)^{a_1+a_2D_q}$
M12	$h = 1.3 + \left(b_1 + \frac{b_2}{DBH}\right)^{-5}$	$h = 1.3 + (H-1.3)\left(1-(a_1+a_2D_q)+\frac{a_1+a_2D_q}{{DBH}/{D_q}}\right)^{-5}$
M13	$h = 1.3 + \frac{b_1}{1+b_2 DBH^{-b_3}}$	$h = 1.3 + \frac{(H-1.3)(1+a_1+a_2D_q)}{1+(a_1+a_2 * D_q)\left({DBH}/{D_q}\right)^{-(a_3+a_4D_q)}}$
M14	$h = 1.3 + \frac{DBH^2}{b_1 + b_2 DBH + b_3 DBH^2}$	$h = 1.3 + \frac{(H-1.3)\left({DBH}/{D_q}\right)^2}{1+(a_1+a_2D_q)\left({DBH}/{D_q}-1\right)+(a_3+a_4D_q)\left(\left({DBH}/{D_q}\right)^2-1\right)}$
M15	$h = 1.3 + \frac{b_1}{1+b_2\exp(-b_3 DBH)}$	$h = 1.3 + \frac{(H-1.3)(1+(a_1+a_2D_q)\exp(-(a_3+a_4D_q)))}{\left(1+(a_1+a_2D_q)\exp\left(-(a_3+a_4D_q)\,{DBH}/{D_q}\right)\right)}$
M16	$h = 1.3 + b_1(1-\exp(-b_2 DBH^{b_3}))$	$h = 1.3 + \frac{(H-1.3)\left(1-\exp\left(-(a_1+a_2D_q)\left({DBH}/{D_q}\right)^{a_3+a_4D_q}\right)\right)}{1-\exp(-(a_1+a_2D_q))}$
M17	$h = 1.3 + b_1(1-\exp(-b_2 DBH))^{b_3}$	$h = 1.3 + \frac{(H-1.3)\left(1-\exp\left(-(a_1+a_2D_q)\,{DBH}/{D_q}\right)\right)^{a_3+a_4D_q}}{\left(1-\exp\left(-(a_1+a_2D_q)\right)\right)^{a_3+a_4D_q}}$
M18	$h = 1.3 + b_1\exp(-b_2\exp(-b_3 DBH))$	$h = 1.3 + \frac{(H-1.3)\exp\left(-(a_1+a_2D_q)\exp\left(-(a_3+a_4D_q)\,{DBH}/{D_q}\right)\right)}{\exp\left(-(a_1+a_2D_q)\exp\left(-(a_3+a_4D_q)\right)\right)}$
M19	$h = 1.3 + \exp(b_1 + b_2 DBH^{b_3})$	$h = 1.3 + (H-1.3)\exp\left(\begin{array}{c}-(a_1+a_2D_q)+\\+(a_1+a_2D_q)\left({DBH}/{D_q}\right)^{a_3+a_4D_q}\end{array}\right)$
M20	$\mathrm{h} = 1.3 + \exp\left(b_1 + \frac{b_2}{DBH+b_3}\right)$	$h = 1.3 + (H-1.3)\exp\left(-\frac{a_1+a_2D_q}{1+a_3+a_4D_q}+\frac{a_1+a_2D_q}{{DBH}/{D_q}+a_3+a_4D_q}\right)$
M21	$h = 1.3 + b_1\exp(-b_2 DBH^{-b_3})$	$h = 1.3 + \frac{(H-1.3)\exp\left(-(a_1+a_2D_q)\left({DBH}/{D_q}\right)^{-(a_3+a_4D_q)}\right)}{\exp\left(-(a_1+a_2D_q)\right)}$
M22	$h = 1.3 + b_1\sqrt{DBH} +$ $+b_2 DBH + b_3 DBH^2$	$h = 1.3 + (H-1.3)\left(\begin{array}{c}\left(1-(a_1+a_2D_q)-(a_3+a_4D_q)\right)\sqrt{{DBH}/{D_q}}+\\+(a_1+a_2D_q)\,{DBH}/{D_q}+\\+(a_3+a_4D_q)\left({DBH}/{D_q}\right)^2\end{array}\right)$
M23	$h = 1.3 + \frac{b_1}{1+(b_2 DBH^{b_3})^{-1}}$	$h = 1.3 + \frac{(H-1.3)\left(1+\frac{1}{a_1+a_2D_q}\right)}{1+\left((a_1+a_2D_q)\left({DBH}/{D_q}\right)^{a_3+a_4D_q}\right)^{-1}}$
M24	$h = 1.3 + b_1 DBH^{b_2 DBH^{-b_3}}$	$h = 1.3 + (H-1.3)\left(\left({DBH}/{D_q}\right)^{(a_1+a_2D_q)\left({DBH}/{D_q}\right)^{-(a_3+a_4D_q)}}\right)$

* Note: DBH is the diameter at breast height, h is the tree height, D_q is the quadratic diameter at breast height in each plot, H is the mean height in each plot, *a* and *b* are model parameters

Table 3. The generalised models from other studies

ID	Model	References
L1	$h = 1.3 + (H - 1.3)\exp\left((a_1 + a_2 H + a_3 D)\left(\frac{1}{DBH} - \frac{1}{D_q}\right)\right)$	Smelko et al. 1987
L2	$h = 1.3 + (H - 1.3)\exp\left(a_1\left(1 - {}^{DBH}/_{D_q}\right)\right)\exp\left(a_2\left({}^{DBH}/_{D_q} - {}^{1}/_{DBH}\right)\right)$	Sloboda et al. 1993
L3	$h = a_1 + a_2 H + a_3 D_q^{0.95} +$ $+ a_4 \exp(-0.08DBH) + a_5 H^3 \exp(-0.08DBH) + a_6 D_q \exp(-0.08DBH)$	Cox 1994
L4	$h = a_1 + a_2 H + a_3 D_q +$ $+ a_4 \exp(a_5 DBH) + a_6 H^{a_7} \exp(a_5 DBH) + a_8 D_q \exp(a_5 DBH)$	Cox 1994
L5	$h = a_1 + a_2 {}^{DBH}/_{D_q} + a_3 H$	Lei et al. 2009
L6	$h = 1.3 + a_1 + a_2 \log\left({}^{DBH}/_{D_q}\right) + a_3 \log(H)$	Lei et al. 2009
L7	$h = 1.3 + \frac{DBH^2(H - 1.3)}{\left(D_q + a_1 H^{a_2}(D_q - DBH)\sqrt{H - 1.3}\right)^2}$	Rymer-Dudzinska 1994, Bruchwald and Wrobelski 1994
L8	$h = H\left(1 - (a_1 + a_2 D_q + a_3 D_q^2) + \frac{a_4 + a_5 D_q + a_6 D_q^2}{{}^{DBH}/_{D_q} + a_{10}} + \frac{a_7 + a_8 D_q + a_9 D_q^2}{\left({}^{DBH}/_{D_q} + a_{10}\right)^2}\right)$	Kuliešis 1989
L9	$h = 1.3 + (H - 1.3)\exp\begin{pmatrix} a_1 \ln D_q + a_2 \ln^2 D_q + a_3 \ln^3 D_q + a_4 \ln DBH + \\ + a_5 \ln^2 DBH + a_6 \ln^3 DBH \end{pmatrix}$	Khlyustov 2015

* Note: DBH is the diameter at breast height, h is the tree height, D_q is the quadratic diameter at breast height in each plot, H is the mean height in each plot, and a_i is the model parameters

Table 4. Model performance criteria selected

ID	Function name	Equation
1	Root mean square error (*RMSE*)	$RMSE = \sqrt{\sum \frac{(y_i - \hat{y}_i)^2}{n}}$
2	Mean absolute percentage error (*MAPE*)	$MAPE = 100 \times \sum \left\lvert \frac{y_i - \hat{y}_i}{y_i} \right\rvert / n$
3	Coefficient of determination (R^2)	$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$
4	Adjusted coefficient of determination (R^2-*adj.*)	$R^2_{adj.} = 1 - (1 - R^2)\frac{(n-1)}{(n-k)}$
5	Akaike information criterion (*AIC*)	$AIC = 2k + n \ln \frac{\sum (y_i - \hat{y}_i)^2}{n}$
6	Bayesian information criterion (*BIC*)	$BIC = k \ln n + n \ln \frac{\sum (y_i - \hat{y}_i)^2}{n}$

* Note: k is the number of model parameters; n is the number of observations; y_i is the measured value; and $\hat{y}_i$ is the predicted value

eters of the trees, six metrics were used: root mean square error (*RMSE*), mean absolute percentage error (*MAPE*), coefficient of determination (R^2), adjusted coefficient of determination (R^2-*adj.*), Akaike information criterion (*AIC*) and Bayesian information criterion (*BIC*). Table 4 summarise the equations of these metrics. Models with the lowest averages of *RMSE, MAPE, AIC* and BIC and with the highest averages of R^2 and R^2-*adj.* are recognized as the best (Aertsen et al. 2010, Ahmadi et al. 2013, Chai et al. 2018). All analyses of data were performed using Python programming language, version 3.5, as well as Pandas, NumPy, SciPy, and scikit-learn software packages (Python 2020, Pandas Development Team 2020, NumPy 2020, SciPy 2020, Pedregosa et al. 2011).

Results

Results of fitting 24 new generalised models and 9 generalised models from other studies are presented in Table 5. Comparison of performance criteria for fitting data and validation data indicates the absence of overfitting for all models. All new generalised models except M12 were well suited to the dataset and accounted for more than 90% of the observed variability (R^2-*adj.*), with *MAPE* values below 8.8%, *RMSE* values less than 1.4 m, and low *AIC* and *BIC* values. Models M9, M14, M20 and M22 do not satisfy the requirement that the height-diameter relationship is given by an increasing function with an upper asymptote. Model M3 has the best quality among generalised models based on two-parameter models (for validation data *RMSE* = 1.145, *MAPE* = 6.613, R^2 = 0.945, R^2-*adj.* = 0.945, *AIC* = 183.4, *BIC* = 192.4). The generalised model M2 based on the Näslund equation showed good quality (for validation data *RMSE* = 1.146, *MAPE* = 6.616, R^2 = 0.944, R^2-*adj.* = 0.944, *AIC* = 183.6, *BIC* = 192.6). Model M24 has the best quality among generalised models based on three-parameter models (for validation data *RMSE* = 1.136, *MAPE* = 6.591, R^2 = 0.944, R^2-*adj.* = 0.944, *AIC* = 176.1, *BIC* = 194.1). In general, differences between performance criteria for different models are often minor.

Among the models L1-L9, only L1 and L7 give the equality of average height and height calculated for a diameter at breast height equal to the quadratic diameter at breast height. L7 model (for validation data *RMSE* = 1.152, *MAPE* = 6.750, R^2 = 0.943, R^2-*adj.* = 0.943, *AIC* = 191.4, *BIC* = 200.3) gives slightly better performance criteria than L1 model (for validation data *RMSE* = 1.164, *MAPE* = 6.860, R^2 = 0.942, R^2-*adj.* = 0.942, *AIC* = 206.5, *BIC* = 220.0). Compared to L7 model, M3 and M24 models achieve the best quality. According to the values of performance criteria, the best of all generalised models is L8 model (for validation data *RMSE* = 0.970, *MAPE* = 5.689, R^2 = 0.960, R^2-*adj.* = 0.959, *AIC* = –20.3, *BIC* = 24.6).

The shape of the curves of the heights and diameters depends on the model (Figure 2). The use of three-parameter base models in the generalized one provides more flexibility for height-diameter curves. All the curves of the dependence of relative height on relative diameter are ordered with one intersection point. With an increase in

Table 5. Performance criteria for generalised height-diameter models for the fitting and validation data

ID	Fitting						Validation					
	RMSE	*MAPE*	R^2	R^2*-adj.*	*AIC*	*BIC*	*RMSE*	*MAPE*	R^2	R^2*-adj.*	*AIC*	*BIC*
M1	1.177	7.066	0.941	0.941	505.0	515.6	1.199	7.089	0.939	0.938	244.3	253.3
M2	1.160	6.713	0.943	0.943	460.4	471.0	1.146	6.616	0.944	0.944	183.6	192.6
M3	1.150	6.760	0.944	0.944	434.6	445.3	1.145	6.613	0.944	0.944	183.4	192.4
M4	1.151	6.773	0.944	0.944	436.1	446.8	1.151	6.653	0.944	0.943	189.4	198.3
M5	1.151	7.738	0.944	0.944	435.8	446.5	1.141	6.607	0.944	0.944	178.7	187.8
M6	1.166	6.773	0.942	0.942	477.6	488.3	1.152	6.656	0.943	0.943	190.7	199.7
M7	1.198	7.274	0.939	0.939	561.2	571.9	1.222	7.304	0.936	0.936	269.5	278.5
M8	1.150	6.750	0.944	0.944	435.0	445.7	1.147	6.646	0.944	0.944	185.3	194.3
M9	1.413	8.806	0.915	0.915	1068.7	1079.4	1.467	8.844	0.908	0.908	510.5	519.5
M10	1.185	6.852	0.940	0.940	526.5	537.2	1.161	6.812	0.942	0.942	201.4	210.4
M11	1.157	6.844	0.943	0.943	453.8	464.4	1.170	6.831	0.942	0.941	211.2	220.2
M12	2.229	14.708	0.789	0.789	2473.1	2483.8	1.153	6.725	0.943	0.943	192.1	201.1
M13	1.149	6.785	0.944	0.944	437.0	458.4	1.147	6.638	0.944	0.944	189.5	207.5
M14	1.149	6.753	0.944	0.944	435.5	456.9	1.137	6.593	0.945	0.945	177.7	195.7
M15	1.185	7.171	0.940	0.940	532.0	553.3	1.177	7.004	0.941	0.941	223.5	241.5
M16	1.151	6.797	0.944	0.944	440.2	461.6	1.142	6.670	0.944	0.944	184.0	202.0
M17	1.152	6.809	0.944	0.943	443.4	464.7	1.144	6.682	0.944	0.944	185.5	203.5
M18	1.172	7.041	0.942	0.941	497.7	519.1	1.162	6.86	0.942	0.942	206.9	224.9
M19	1.176	7.078	0.941	0.941	507.9	529.3	1.199	7.087	0.939	0.938	248.4	266.3
M20	1.153	6.807	0.944	0.943	447.0	468.4	1.144	6.629	0.944	0.944	186.4	204.4
M21	1.146	6.716	0.944	0.944	426.5	447.9	1.144	6.584	0.944	0.944	185.7	203.7
M22	1.157	6.883	0.943	0.943	458.2	479.6	1.152	6.767	0.943	0.943	195.5	213.4
M23	1.150	6.755	0.944	0.944	437.4	458.8	1.147	6.668	0.944	0.943	189.9	207.8
M24	1.145	6.722	0.944	0.944	425.0	446.4	1.136	6.591	0.945	0.945	176.1	194.1
L1	1.185	6.917	0.940	0.940	530.0	546.0	1.164	6.860	0.942	0.942	206.5	220.0
L2	1.423	8.915	0.914	0.914	1090.5	1101.2	1.498	8.965	0.904	0.904	538.3	547.3
L3	1.078	6.589	0.951	0.950	242.8	274.9	1.064	6.467	0.952	0.951	93.7	120.6
L4	1.285	7.927	0.930	0.930	787.9	830.6	1.339	8.019	0.924	0.923	401.6	437.6
L5	1.384	9.330	0.919	0.919	1006.2	1022.2	1.398	9.301	0.917	0.916	448.9	462.4
L6	1.673	11.431	0.881	0.881	1590.7	1606.7	1.670	11.14	0.881	0.880	684.3	697.8
L7	1.160	6.764	0.943	0.943	462.2	472.9	1.152	6.750	0.943	0.943	191.4	200.3
L8	1.012	5.893	0.957	0.956	55.5	108.8	0.970	5.689	0.960	0.959	–20.3	24.6
L9	1.063	6.467	0.952	0.952	201.1	233.1	1.054	6.449	0.953	0.952	81.6	108.6

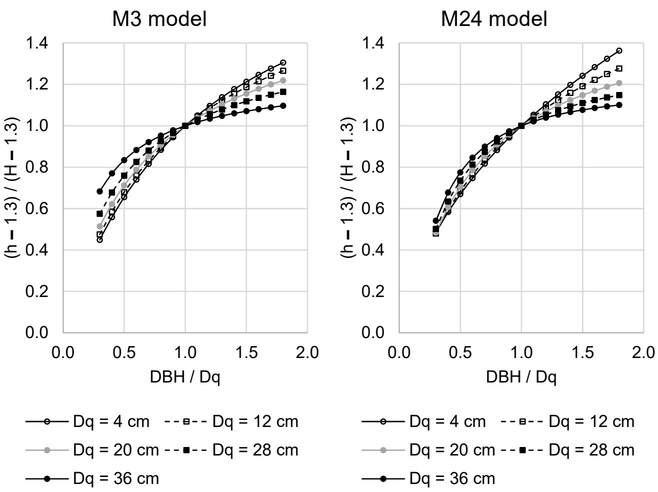

Figure 2. M3 and M24 model prediction relationship of relative heights to relative diameters for quadratic diameter at DBH from 4 to 36 cm

quadratic diameter at breast height during the growth of stands, there is a change in the relationship between relative heights and relative diameters. In mature stands the curve is more convex than in young stands.

Three-parameter models are more flexible than two-parameter models and allow for more detailed transfer of dependencies. With many observations on trial plots, three-parameter models give a good result. Plots of residuals in the fitting and validation phase of M24 model are shown in Figure 3. A large deviation in the residuals was seen only for a few trees, which were caused by extreme outlier observations. QQ-plot of the standardised residuals showed the normal distribution pattern. This indicates significant skewness were absent in the residuals. The location of the residuals on the graph shows the lack of autocorrelation. Our residual plots are consistent with generalised model selections in other studies (Sánchez-González et al. 2007, Ahmadi and Alavi 2016).

Therefore, the final generalised height-diameter model (M24) adapted to all data was:

$$h = 1.3 + (H - 1.3)\left(\left(\frac{DBH}{D_q}\right)^{(0.597-0{,}0111D_q)\left(\frac{DBH}{D_q}\right)^{-(-0{,}112+0{,}0283D_q)}}\right),$$

where:
DBH – the diameter at breast height (cm),
h – the tree height (m),
D_q – the quadratic diameter at breast height (cm),
H – the mean height (m).

The resulting model is continuous concerning the quadratic diameter at breast height and average heights, giving it an advantage over height class tables used in Russia. Such a model gives curves of the dependence of heights on diameters for stands of all combinations of quadratic diameter at breast height and average heights regardless of age, growing conditions, or geographical area (Kuliešis 1989). It is essential to develop specific equations for each species because each one has particular growth habits. Additionally, these types of equations facilitate the quantification of existing timber forest resources (Santiago-García et al. 2020).

Developing a simple and accurate height-diameter model makes it possible for model users to predict tree heights by relying on measurements of DBH and other covariate predictors. They are derived from forest inventory databases. The existing generalised height-diameter models (Kuliešis 1989, Khlyustov 2015) for the birch stands in European Russia with the same set of variables have many parameters. Our model with 4 evaluated parameters is of acceptable quality. Our model will be useful for the inventory crew, who may measure the heights of only a few trees per plot and predict the heights of the remaining trees using this model.

Conclusions

The variables diameter at breast height, quadratic diameter at breast height, and mean height proved to be the suitable variables to predict trees height. The models showed a good predictive performance, and their ease of application constitutes one of the main advantages of the present models. Significantly, it is easily implementable in forest inventory procedures or growth simulators. Results show that there existed little differences between models. The performance statistics showed that modified power function the most suitable and recommended for predicting the height-diameter relationships for birch trees in European Russia. The methodology of the study allows the similar work for tree species and forest conditions, for which information about the nature of the relationship of height with the diameter at breast height is incomplete or absent.

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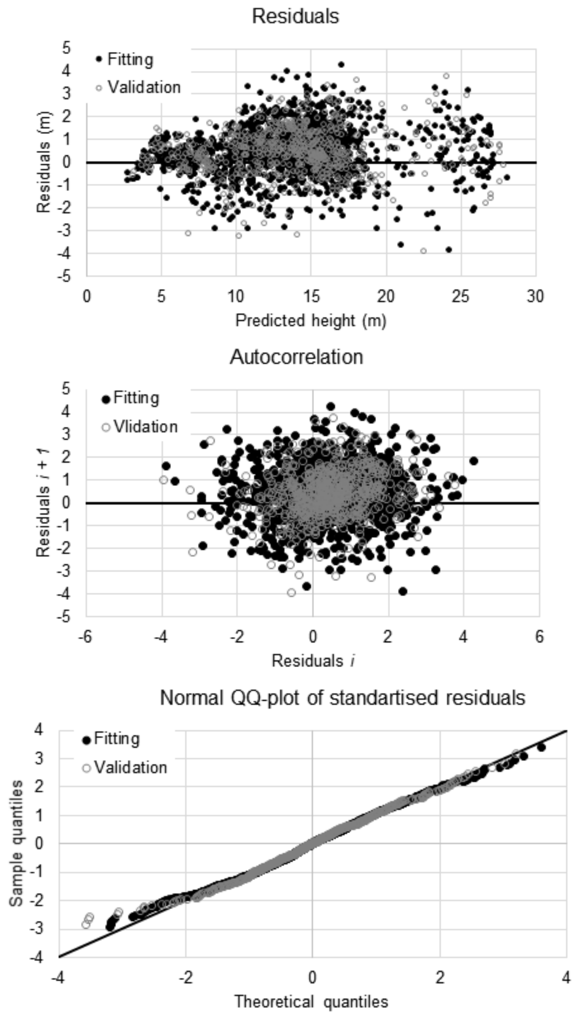

Figure 3. Residual plots for M24 model

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